

## Spatio-Temporal Patterns of a Plastic Network System

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Spatio-temporal patterns of two-dimensional plastic networks have been studied numerically. Each constituent element evolves following a threshold dynamics and the interactions between elements are temporally changed according to the current states of elements. Such a network system is highly adaptive and thus it is expected to be useful for a model to describe central nervous systems. The evolution of the system from disordered states yields cyclic motions spatially separated. These phenomena may give an example of temporal order.

### § 1. Introduction

Non-equilibrium statistical physics has made a great progress and fields covered by it have been spreading very rapidly up to living systems.

Neural networks are quite interesting systems to be studied from a viewpoint of non-equilibrium statistical physics. In the traditional physics, interactions are assumed to be time-independent. In order to study cooperative phenomena of living systems, it is essential to introduce "time-dependent" interactions among elements in living systems, as was emphasized by one of the present authors in a specific model.<sup>1)</sup> Such time-dependent interactions may be called to be "*plastic*".

In the present paper,<sup>\*</sup> we introduce a typical "*plastic cellular automata (CA)*"<sup>2)</sup> as a model of neural systems. Usually, in modeling central nervous systems (CNS), information put into the relevant system is symbolized using spatio-temporal patterns of neural firing. That is, spatial locations of neurons in active states and temporal behaviors of neurons (firing rate of neurons) would play a role of carriers of information in the system. However, there have been reported very few studies concerning its firing spatial patterns and most of previous works have been performed on its statistical properties (such as the number of firing neurons).<sup>3),4)</sup>

We study here especially on its dynamical patterns. A real CNS is characterized by its plastic structure<sup>5)</sup> constantly tuned by external and internal signals. In the field of the brain research, the emphasis has been given on elucidating a mechanism of "*plasticity*" of neural network. Namely, the mechanism which tunes an effectivity of transmitting signals among neurons. Recently, tuning mechanism between two neurons has been elucidated gradually.<sup>5)</sup> However, what kinds of dynamics or patterns are expected to be observed in the network is still missing, and further, how the plasticity works towards storing memory is completely unapprehended. The purpose of the present paper is to study such a network consisting of modifiable wires (synapses) and elements (neurons). Such a plastic structure is a necessary factor for the system to possess an adaptability to its surroundings.<sup>6)</sup>

To clarify and embody the word "*plastic*", some basic terms are introduced in

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§ 2. In § 3, simulations of a plastic network are presented. In §§ 4 and 5, we analyze the mechanism which causes a cyclic motion. We report here several typical spatial patterns which give a long cyclic motion mostly independent of network parameters.

## § 2. Basic modeling of neural network

### 2.1. States of neural network

A central nervous system (CNS) is a network of interconnected neurons. Connections between neurons are called “*synapses*”, and neurons are interacting with each other by sending signals in electrical and chemical forms through these synapses.

Several assumptions have been proposed for constructing models for CNS.<sup>5)-7)</sup> “*All or none principle*” is the first assumption. The principle is as follows: At a given time  $t$ , each neuron is either in a firing state or a non-firing state and no intermediate state exists. A “*firing state*” is an active state in which a neuron can transmit signals to coupled neurons. Analogously to the Ising spin system, we assign to each neuron (at time  $t$ ) following two-valued state-variables:

$$S_x(t) = \begin{cases} 1 & \text{(firing state),} \\ -1 & \text{(non-firing state).} \end{cases} \quad (2.1)$$

A signal is transmitted from “ $j$ -th neuron” to “ $i$ -th neuron” in forms of electrical pulses and chemical substances through synaptic connections which are expressed as

$$\sum_j J_{i \leftarrow j} (S_j + 1) / 2 \quad (2.2)$$

in our models where  $J_{i \leftarrow j}$  represents the strength of coupling.  $J_{i \leftarrow j}$  can be either positive or negative corresponding to excitatory or inhibitory signal, respectively. An excitatory signal is positively integrated and thus works as a factor to fire the neuron which receives it, whereas an inhibitory signal has an opposite role.

The state of a network at time  $t$  is described by two qualitatively different types of variables as

$$\Phi(t) = (S_1(t), S_2(t), \dots, S_N(t)), \quad (2.3)$$

$$\Psi(t) = (J_{11}(t), J_{12}(t), \dots, J_{NN}(t)). \quad (2.4)$$

The first one is binary sequences of neural states and the second one is the set of strength of synapses.

In dealing with CNS as a deterministic dynamical system of a discrete time, a relevant mapping from  $\Phi(t)$  and  $\Psi(t)$  into  $\Phi(t+1)$  and  $\Psi(t+1)$ , respectively, is introduced. This is one of the finest forms to describe an evolutionary way of the network. Hereafter we restrict ourselves to a discrete time ( $t \rightarrow n$ ) and discrete space dynamics.

We introduce here some assumptions peculiar to our model. We take such a mean-field-like treatment that a sign of interaction is determined by a receptor side (arrow-head side) of interaction. By this treatment, we lose information about which signal is positive or negative, but only its integrated value is retained. If the integrat-

ed value exceeds a threshold, a neuron receiving signal fires (namely, transmits a signal to coupled neurons). Otherwise, it is eliminated.

For simplicity, the threshold value  $H_x$  is taken to be proportional to its synaptic strength.

i)  $J_{x \rightarrow y} = J_x$ ;  $J_{x \rightarrow y}$  only depends on site  $x$ .

ii)  $H_x = V \cdot J_x$ ; the threshold value is proportional to  $J_x$ .

By these assumptions, synapses become site-random fields and (2.4) is reduced to (2.5).

$$\Psi(n) = (J_1(n), J_2(n), \dots, J_N(n)). \quad (2.5)$$

It should be noted that the asymmetrical condition ( $J_{x \rightarrow y} \neq J_{y \rightarrow x}$ ) is still kept in (2.5).

## 2.2. Dynamics of states

Each firing decision of a neuron is determined by a threshold mechanism. These neurons are able to change their states synchronously, spontaneously, or in other ways. We express these "scanning" methods by introducing the following "scanning operator"  $T_i$  ( $i=1, 2, \dots, n$ ) where  $T_i$  operates on the  $i$ th site:

$$S_x(n+1) = T_x S_x(n). \quad (2.6)$$

Generally, the  $T_x$ 's are non-commutative operators ( $T_1 T_2 \neq T_2 T_1$ ). More explicitly,  $T_x$  in (2.6) is defined as follows:

$$\text{if } \sum_y S_y(m) > V; S_x(n+1) = \text{sgn}(J_x(n)), \quad (2.7)$$

$$\text{otherwise } ; S_x(n+1) = -\text{sgn}(J_x(n))$$

$$\text{with } \text{sgn}(x) = \begin{cases} 1 & (x > 0), \\ -1 & (x \leq 0), \end{cases} \quad (2.8)$$

where  $m=n+1$  if  $T_y$  operates before  $T_x$  and  $m=n$  otherwise. The symbol  $\sum_y$  denotes the summation over all neurons coupled to  $x$  and  $V$  is the threshold value at site  $x$ .

By such a mean-field-assumption, a neuron which receives signals from coupled neurons will follow the induced mean-field  $\text{sgn}(J_x(n))$ , if the number of coupled neurons in an active state exceeds the threshold value  $V$ .

Typical three scanning methods are listed below:

TYPE 1 (Synchronized way);  $T_y$  operate simultaneously on all the sites.

TYPE 2 (Spontaneous way);  $\prod_j T_j = T_N T_{N-1} T_{N-2} \dots T_1 T_3 \dots$ , etc.

Operating orders are chosen randomly.

TYPE 3 (Regular way);  $\prod_j T_j = T_N T_{N-1} T_{N-2} \dots T_2 T_1$ .

$T_y$  operate in order from the 1st site to the  $N$ th site.

A mapping of network state  $\Psi(n)$ , namely an evolutionary way of distribution form of synapses, should be carefully determined. One of the most fundamental forms comes from Hebb's hypothesis.<sup>10)</sup> His original idea of the hypothesis is that the

coupling coefficient  $J_{x \rightarrow y}(n+1)$  between two neurons ( $x$  and  $y$ ) is increased when  $S_x(n+1)=1$  and  $S_y(n)=1$ .

Here the Hebb hypothesis is modified and is extended to a more symmetrical form. That is, if  $\text{sgn}\{S_x(n)\sum_y S_y(m)\}=1$  then  $J_x(n)$  is increased and, it is otherwise decreased. A momentary pre-postsynaptic correlation ( $S_x(n)\sum_y S_y(n)$ ) determines the tuning mechanism of the strength of synapses in a synchronous scanning type. By using the scanning method defined previously,  $J_x(n)$  is mapped into  $J_x(n+1)$  as

$$J_x(n+1) = T_x J_x(n). \quad (2.9)$$

For our numerical calculation, the following functional form is used in the present paper. This is a natural extension of the so-called Cooper-construction:<sup>11)</sup>

$$\begin{aligned} T_x J_x(n) &= F(J_x(n)) + G(\{S\}), \\ F(J) &= (1 - \gamma)J, \\ G(\{S\}) &= \alpha_1 S_x(n) \sum_y S_y(m) - \alpha_2 \langle S_p S_q \rangle^3 \end{aligned} \quad (2.10)$$

with some constants  $\gamma$ ,  $\alpha_1$  and  $\alpha_2$ , where

$$\langle S_p S_q \rangle \equiv \sum_x S_x(n) \sum_y S_y(n) / N. \quad (2.11)$$

(average value of a correlation at time  $n$ )

### § 3. Simulations

#### 3.1. Attractors

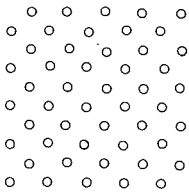
A neural network has its neurons in a rigid spatial site. Indeed, in a cerebral cortex of a mammalian, neurons are not randomly distributed but they organize a regular network structure.<sup>5),7)</sup> We use here a square lattice network of 100 sites with a periodic boundary condition in which each neuron is assigned in a two-dimensional plane and couples to its four nearest neighbors. In this square lattice network, a threshold value  $V=3$  would be a value favorable for causing a cyclic motion as is shown in the next subsection.

The asymptotic behavior of this network can be classified roughly into three categories from its spatio-temporal patterns, essentially in the same way as Wolfram classified in cellular automata (see Fig. 1).<sup>2),12),13)</sup>

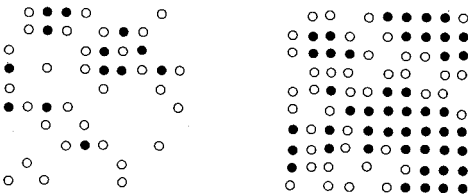
- 1) The same firing pattern is repeated for all time steps.
- 2) Finite numbers ( $\geq 2$ ) of  $\Phi(n)$  and  $\Psi(n)$  are repeated,
  - i) spatially localized periodic structure corresponding to fixed points,
  - ii) catastrophic structure which is periodic but not spatially localized.
- 3) A localized aperiodic structure survives, or such an aperiodic region intrudes into other neighboring regions.

It should be noted that the recurrence of the same firing patterns does not necessarily mean any more that the system enters into cyclic modes for a plastic network.

The occurrence of a cyclic mode with periodicity  $K$  is identified when the next conditions are satisfied:

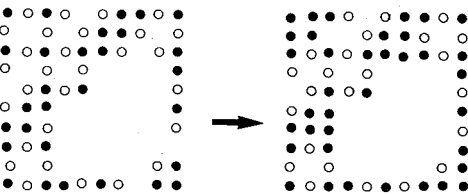


- 1) Fixed point-like-attractor: no eligible neurons exist.



- 2) i) Spatially localized oscillating areas coexist ( $K=84$ ). (right figure)

ii) Catastrophic structure; almost all the oscillating areas are connected ( $K=2$ ). (left figure)



- 3) Neighboring epileptic or dead neurons (left figure) become eligible after 1000 times steps (right figure).

Fig. 1. Phenomenological classification of network behaviors in an attractor region, except for 3) which is assumed to be in a long transient region. Each site with eligible, epileptic and dead neurons is represented by black point, white point and blanks, respectively. A simulation is performed on a square lattice of 100 sites. The initial strength of synapse at each site is taken to be random, namely, takes an independent value from  $[-1, 1]$  with equal probability. The initial state of a neuron at each site is taken as  $S_x(0) = -\text{sgn}(J_x(0))$  (e.g., values 1 or  $-1$  independently).

$$S_x(n+K) - S_x(n) = 0$$

and

$$|J_x(n+K) - J_x(n)| < \delta (\sim 0.001) \quad (3.1)$$

for all sites  $x$ .

Here the step number  $K$  is a minimal length of the recurrence of the same state. We identify  $K$  with a periodicity of the cyclic mode of an attractor and will be called either a reducible or irreducible cycle. A reducible cycle is defined as a superposition of subcycles  $(K_1, K_2, K_3, \dots)$ . In such a case,  $K = \text{L.C.M. of } (K_1, K_2, K_3)$ . On the other hand, an irreducible cycle cannot be decomposed into subcycles (see the next section).

Each neuron is also categorized into the following three types according to its behaviors as Clark et al.<sup>5)</sup> discussed in their paper:

- 1) dead neuron...neuron which never fires,
- 2) epileptic neuron...neuron which always fires (shows cyclic behavior, but its period of cycle is the same as in units of discrete time steps),
- 3) eligible neuron...neuron which is not locked into either extreme behavior.

### 3.2. Firing loops

The purpose of this simulation is to find the linked stable firing loop of eligible neurons for varying the relevant parameters and initial conditions. A definition of loop is that each eligible neuron couples to at least one eligible neuron in its nearest neighbors.

It is widely believed that there are two different kinds of mechanism that












correspond to a short term and a long term memory.<sup>6)~9)</sup> The first idea is to relate those loops to memory carriers. It is a natural and fascinating idea to imagine some relations between "associative memories" or "thought" and "loop" or "loop-loop coupling", respectively.

A computer simulation has shown the existence of firing loops but with a fairly different mechanism in comparison with that of a static network system (a system with fixed values of synapses).

In a static network system, the strength of synapses is fixed but their values are randomly distributed. In a two-dimensional square lattice network with a threshold value 3, numerical simulations show that most of attained periods of cyclic modes are 4 for scanning types 1, and 2 for type 3. It should be noted that more than half of the initial states go to a fixed point attractor (no oscillating behavior).

On the contrary, most of the attained attractors show a cyclic motion (namely

Table I. Examples of cyclic modes. Each cycle is decomposed into simple and rich subcycles. In the notation  $M^a_b$ ,  $M$  denotes the period,  $a$ , the number of the same pattern and  $b$ , the number of eligible neurons constructing the cyclic motion. Spatial patterns of rich subcycles are also depicted.

Period	Decomposition into subcycles	Number of eligible neurons
Scanning type 1 (Parallel)		
12	$12^1_3$ (  ) $\otimes 6^6_1 \otimes 1^{29}$	9
66	$66^1_4$ (  ) $\otimes 6^7_1 \otimes 1^{29}$	11
462	$77^1_2$ (  ) $\otimes 7^2_3$ (  ) $\otimes 6^5_1 \otimes 1^{31}$	13
Scanning type 3 (Sequential)		
12	$6^6_1 \otimes 4^1_1 \otimes 1^{39}$	7
18	$9^1_4$ (  ) $\otimes 9^1_2$ (  ) $\otimes 6^9_1 \otimes 1^{33}$	15
66	$66^1_5$ (  ) $\otimes 6^4_1 \otimes 1^{30}$	9
84	$12^1_3$ (  ) $\otimes 7^1_3$ (  ) $\otimes 6^7_1 \otimes 1^{29}$	13
288	$32^1_3$ (  ) $\otimes 9^3_2$ (  ) $\otimes 6^6_1 \otimes 1^{29}$	15

class 2 behavior by the previous definition) in this plastic network system. It is found that a periodicity  $K$  of the network is L.C.M. of each period of these localized periodic structures.

The total cyclic mode  $K$  will be decomposed into

1)  $\otimes$  (simple subcycles)

and

2) (simple subcycles)  $\otimes$  (rich subcycles),

where the symbol  $\otimes$  represents a direct product (see Table I).

A simple subcycle is always a simple irreducible one which only one eligible neuron participates in the cyclic mode (this is not a loop by definition). A spatially localized pattern of a simple subcycle is an eligible neuron surrounded by three epileptic neurons and one dead neuron. In such a case, a possible periodicity of simple subcycle is 2, 4 and 6.

On the other hand, the cyclic mode of the whole network with periodicity  $K$  which is larger than  $12(= \text{L.C.M. of } (6, 4, 2))$  should contain rich subcycles. Rich subcycles which contain more than two eligible neurons are composed of minimum firing loops by definition.

For random initial conditions, well observed spatially localized patterns are shown in Fig. 2. Although the periods of cycle they carry depend on the network parameters ( $\gamma, \alpha_1, \alpha_2$ ), and also on the location of epileptic and dead neurons surrounding the eligible neurons, these geometrical patterns are scanning-type independent.

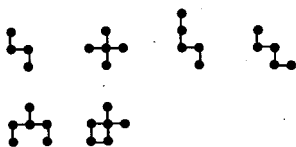
As it is depicted in Figs. 2 and 3, these rich subcycles are "twisted strings" which consist of nearly one-dimensional coupled eligible neurons. In the case of one-dimensionally connected case (e.g., each neuron in rich subcycles has exactly two coupled eligible neurons except the edging neuron of a string), we can assign the numbers from one end of a string to the other. Such rich subcycles are able to be compared with one-dimensional CA by writing a spatio-temporal pattern.

Although some patterns are similar to that of one-dimensional CA,<sup>(12),(13)</sup> the mechanisms that cause those patterns are different. In one-dimensional CA, particular rules create propagating patterns, on the other hand, spatial inhomogeneity of synaptic distribution generates such patterns in our model.

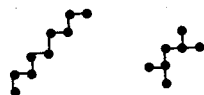
Another important aspect of rich



1) Geometrical patterns of most probable rich sub-cycles.



2) Uncommon but observed patterns.



3) Patterns generated from a special initial state.

Fig. 2. The total period  $K$  will be decomposed into products of subcycles. A rich subcycle is defined as a subcycle containing more than two eligible neurons. Examples of spatial patterns of these rich subcycles are listed as in 1)~3).

subcycles is that there exist particular eligible neurons which simultaneously cooperate in more than two subcycles. Some of obtained large subcycles can be taken as linking of smaller size of rich subcycles.

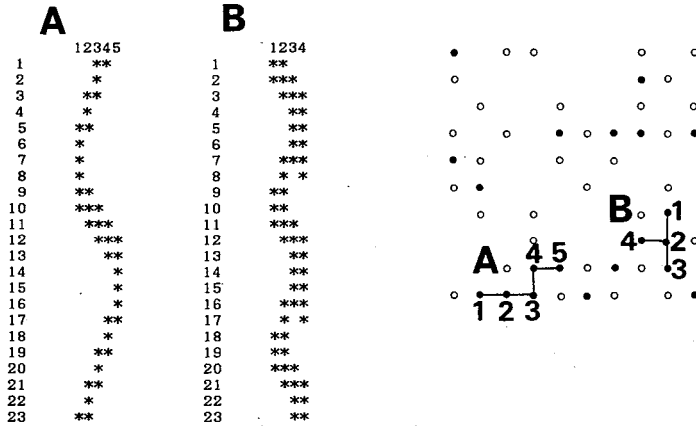


Fig. 3. Space-time section of rich sub-cycles. After a network is detected to a cyclic motion, oscillating areas are picked out and the constituent neurons are numbered and expanded horizontally. The configurations of successive steps are shown on successive horizontal lines. Neurons on firing states are represented by stars, and non-firing states by blanks. Period  $K$  equals 18 for  $A$  and  $K$  equals 9 for  $B$ . The initial state of a neuron at each site is taken as  $S_x(0) = -\text{sgn}(J_x(0))$  (e.g., values 1 or -1 independently).  $\gamma = 0.05$ ,  $\alpha_1 = 0.005$ ,  $\alpha_2 = 0.005$  and a scanning type 3.

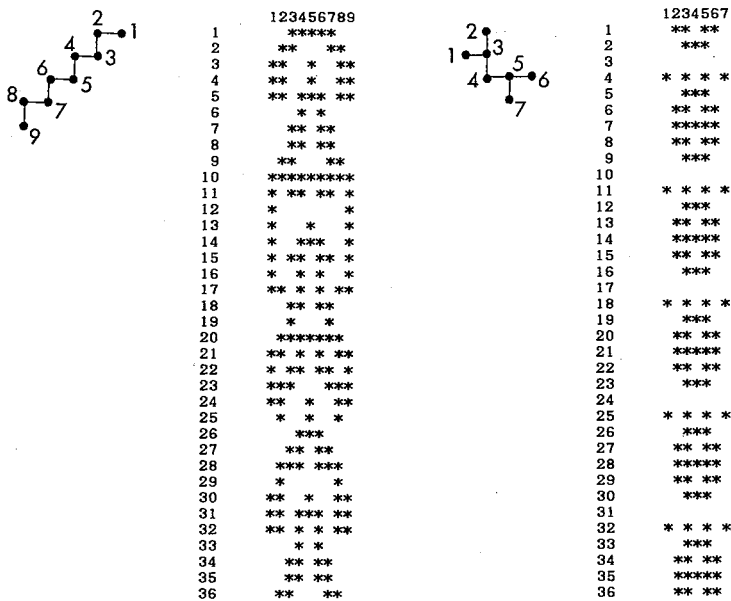




Fig. 4. Space-time sections of obtained large local cyclic motion are depicted (notations: see the captions of Fig. 3). It shows that processing signals (firing state) propagate symmetrically relative to the central neuron. The period  $K$  equals 576 (right) and  $K$  equals 84 (left). The initial states of neurons of all sites are taken as  $S_x(0) = 1$  and  $J_x(0) = 1.0$ .  $\gamma = 0.05$ ,  $\alpha_1 = 0.005$ ,  $\alpha_2 = 0.00053$  (right figure) and  $\alpha_2 = 0.00062$  (left figure). The scanning type 1 is used in the above two figures.



Examples of such large subcycles containing 9 and 7 eligible neurons are depicted in Fig. 4. It has a symmetrical form and the processing signals (sites of neurons in a firing state) propagate symmetrically to both the ends of a string. Namely, a central neuron in these subcycles links two rich subcycles (  ) and (  ), respectively.

Unfortunately, these large subcycles are uncommon for systems starting from a random initial state and can be obtained under setting special initial conditions.

### § 4. Mechanism of subcycles

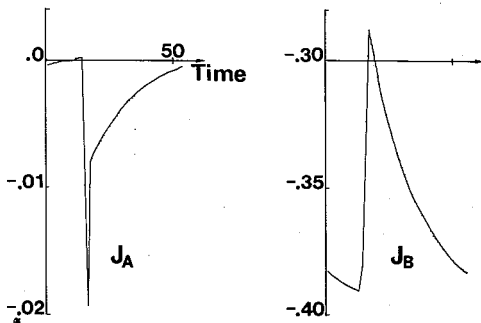
Mechanism which produces subcycles is due to the dynamical behavior of  $J_n$ . Basically, the mechanism which causes a cyclic motion (for a threshold value  $V=3$ ) is that if one non-active neuron ( $S_x$ ) is surrounded by three epileptic and one dead neurons, then  $J_x$  should gradually change its value towards firing neuron ( $S_x$ ). Starting with a randomly distributed initial state, the average value of correlation  $\langle S_p S_q \rangle^3$  (the third term in  $J_x$ ) is negligible in the attractor region and is approximated by a small constant value  $\epsilon$ .

A maximum period of a simple subcycle is  $K=6$ . Namely, the states of a neuron change every three steps (see Appendix A). On the contrary, rich subcycles are brought about, when the surrounding neurons contain eligible ones. In such a case, the situation is not simple enough to analyze its mechanism.

The following five different mappings are possible:

$$J_x(n+1) = (1 - \gamma)J_x(n) + \alpha_1 C - \epsilon \tag{4.1}$$

with  $C = -4, -2, 0, 2, 4$ . These correspond to  $I_1, I_2, I_3, I_4, I_5$ . They differ in the second term which is the summation of states of coupled neurons. The temporal evolution of  $J$  equals the temporal ordering of these mappings. (For example,  $I_2 \rightarrow I_5 \rightarrow I_3 \rightarrow I_1 \rightarrow I_4 \rightarrow I_1 \rightarrow \dots$ ).

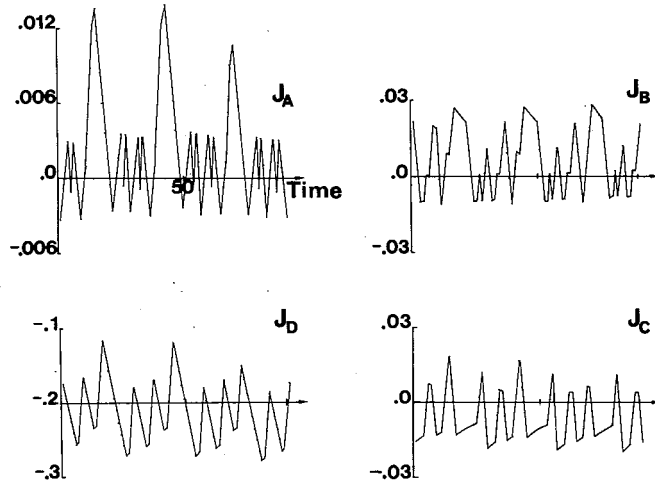


Type-I) A rich subcycle with period 54. Two eligible neurons surrounded by five epileptic and one dead neurons form a loop.  $\gamma = 0.05$ ,  $\alpha_1 = 0.005$ ,  $\alpha_2 = 0.005$ ,  $\langle \epsilon \rangle = -0.0073$ .

Fig. 5. (continued)

These orderings are determined by the states of the surrounding neurons (e.g., the second term  $C$ ). Inversely, the phase difference of a neighboring  $J$  determines the temporal evolution of neurons.

Two types of long cycle are observed in a rich subcycle (see Fig. 5). One is caused essentially by smallness of  $\epsilon$  (type I) and the other is independent of  $\epsilon$  (type II). For the former type, even two eligible neurons show a long periodic behavior. The mapping  $I_3$  (namely  $C=0$ ) is dominant, which causes long iteration steps to change the sign of  $J_x(n)$ . For example, the time evolution of  $J_A$  in



Type-II) A rich subcycle with period 90. Four eligible neurons surrounded by seven epileptic and one dead neurons form a loop.  $\gamma=0.05$ ,  $\alpha_1=0.005$ ,  $\alpha_2=0.00068$ ,  $\langle \epsilon \rangle = -0.0000031$ .

Fig. 5. Showing time evolution of  $J_s$  which participate in a cyclic motion. Abscissa and ordinate represent a value of  $J$  and time steps, respectively. There are two types of long cyclic motion. (Types I and II) We use a scanning type 1 in these examples.

Fig. 5(type I) is given by the following iteration sequence per one cycle,

$$(I_2)^2(I_4)(I_3)^{51}. \quad (4.2)$$

As  $\epsilon$  decreases, a longer period should be attained. The mapping  $I_3$  can be approximated by a differential equation over  $n$  and a rough estimate shows that a number of iterations needed to change the sign of  $J_x(n)$ , which nearly equals a period of the subcyclic motion, is given by

$$t \sim \gamma^{-1} \log |\gamma J_0 / \langle \epsilon \rangle|, \quad (4.3)$$

where  $J_0$  is the maximum value of  $J_x(n)$  and  $\langle \epsilon \rangle$  is an averaged value of  $\epsilon$  in the attractor region.

The latter type is essentially due to the multiple spatial connection among neurons. Mappings with  $C \neq 0$  are dominant, and consequently  $J_x(n)$  changes its sign easily. The time evolution of  $J_A$  in Fig. 5 (type II) is given by the following iteration sequence per one cycle,

$$(I_4)(BCD)^2(I_4)^3(I_5)^2(I_2)^6(CD), \quad (4.4)$$

where

$$B=(I_4)^3(I_5)^3(I_2)^3, \quad C=(I_4)^3(I_2 I_5 I_1 I_2 I_5) \quad \text{and} \quad D=(I_4)^2(I_2 I_5 I_1)(I_2)^2.$$

The propagation of patterns is observed in this case.

Rich subcycles are unstable against the noise (a stochastic flip of a single neural site). On the other hand, simple subcycles are not so. This is because a rich subcyclic motion comes from a subtle phase difference of the  $J$ 's between coupled neurons,

whereas simple one can oscillate by itself.

In the above discussions, the third term was taken as a small constant only in the attractor region. The whole dynamical behavior becomes, however, much different, if it is assumed to be constant from the beginning. The third term oscillates and its amplitude is large in initial and transient processes of the system. For example, when the third term exceeds the maximum value of the second term, the system suddenly falls into a catastrophic behavior (class 2-(ii)). Most of the catastrophic behaviors are due to the sudden growth of the third term.

## § 5. Discussion

In the present paper, we have investigated the spatio-temporal behavior of plastic network systems in two dimensions from a phenomenological point of view. Our first aim was to find a long and stable loop of eligible neurons using plasticity. Cyclic behaviors obtained are decomposed into rich and simple subcyclic motions and the former subcycle is indeed a string of eligible neurons (which is called a "loop" here).

The attained loops are grouped into typical localized geometrical patterns. Each loop has its own periodicity according to the spatial location of surrounding epileptic and eligible neurons. Geometrical patterns of these loops are twisted strings and some of them are one-dimensionally connected. A space-time section of loop shows a behavior qualitatively similar to the section of one-dimensional CA.<sup>12)</sup> Especially, breathing patterns are worth noting to study. The length of one pitch of breathing motion is determined from the phase difference of synapses participating in the motion.

Though the simulation was performed on a small system, periods of subcycles are expected to be the same even in larger systems. Because mechanisms of each subcycle are due to a local property of the system. In a large system, there is a possibility to have a firing loop of a large population of eligible neurons.

In recent years, lots of works have been published on problems of memory storing from the viewpoint of spin glass models.<sup>3),11),14),15)</sup> By the term "memory", we restrict here, patterns being put initially in the net and actually becoming attractors under kinetics of each element. Attractors of spin glass models are fixed points and cyclic behaviors are infrequently observed. The advantage of the Hopfield spin glass model is that spatial patterns of attractors can be expressed in terms of the distribution of synapses. The problem is how to control the basin structure of these attractors. It is to make most of the initial states go to "wanted attractors" and to make the rest of initial states go to "unwanted attractors".

Up to now, a capacity of memory storing attributes to a static distribution of synapses. In an actual brain system, however, the distribution is dynamical and not static. We believe that even if it is possible to store memory in a static system, such a mechanism may be quite different from the true mechanism of actual systems. To understand the real system, the meaning of memory should be changed drastically. Our optimistic idea is to seek the meaning in the phenomena we have discussed. Namely, patterns appear recurrently and compete with or stabilize each other.

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Numerical calculations were performed by FACOM M190 and by NEC-9801 (Micro-computer).

### Appendix A

#### — Periodicity of a Simple Subcycle —

Assuming the value ( $\sum S_y$ ) of neurons  $\{S_y\}$  coupling to  $x$  equals 2, the following processes are possible as seen from (2.6):

	$(S_x(n), J_x(n))$	$\rightarrow$	$(S_x(n+1), J_x(n+1))$
1)	(1, positive)	$\rightarrow$	(-1, $aJ_x(n)+b$ )
2)	(1, negative)	$\rightarrow$	(1, $aJ_x(n)+b$ )
3)	(-1, positive)	$\rightarrow$	(-1, $aJ_x(n)-b$ )
4)	(-1, negative)	$\rightarrow$	(1, $aJ_x(n)-b$ ),

where  $a=1-\gamma$ ,  $b=2a_1$  and  $\varepsilon$  is neglected (see Eq. (4.1)). A periodic orbit is attained by the following closed sequence

$$2) \rightarrow 1) \rightarrow 3) \rightarrow 4) \rightarrow 2) \rightarrow \dots,$$

being classified above with allowed repeat of each process.

Starting with a negative  $J_x(n)$  with  $S_x=1$ (process 2),  $J_x(n)$  increases until it becomes positive. Hereafter  $J_x(n)$  is simply denoted by  $J_n$ .

Sequence 1;  $J_1 \rightarrow J_2 \rightarrow J_3 \rightarrow J_4 \rightarrow J_5 \rightarrow \dots J_n$

$$J_{n+1} = a^n J_1 + b(1 - a^n)/(1 - a). \tag{A.1}$$

On the other hand, with a positive  $J_x(n)$  with  $S_x=-1$  (process 3),  $J_x(n)$  decreases until it becomes negative.

Sequence 2;  $J'_1 \rightarrow J'_2 \rightarrow J'_3 \rightarrow J'_4 \rightarrow J'_5 \rightarrow \dots J'_n$

$$J'_{n+1} = a^n J'_1 - b(1 - a^n)/(1 - a). \tag{A.2}$$

Assuming that the number of steps is  $n$  in (A.1) and (A.2), respectively, we put  $J_1 = J'_{n+1}$  and  $J'_1 = J_{n+1}$  to find a closed orbit:

$$J'_k = a^{2n} J'_{k-1} - b(1 - a^{2n})/(1 - a). \tag{A.3}$$

By a sufficiently large number of iterations where the parameter  $k$  is used to specify each  $2n$  iterations,  $\{J'_k\}$  converge to

$$\lim_{k \rightarrow \infty} J'_k = -(1 - a^n)b/[(1 + a^n)(1 - a)]. \tag{A.4}$$

The obtained periodic orbit consisting of  $2n$  points is given by

$$J_k^{(\pm)} = \pm b / (1 - a) \mp 2ba^{k-1} / [(1 + a^n)(1 - a)], \quad (k=1, \dots, n) \quad (\text{A}\cdot\text{5})$$

whereas the threshold rule requires the following conditions ( $n \geq 2$ ):

$$J_{n-1}^{(+)} < 0 \quad (J_{n-1}^{(-)} > 0) \rightarrow 1 + a^n < 2a^{n-2}, \quad (\text{A}\cdot\text{6})$$

$$J_n^{(+)} > 0 \quad (J_n^{(-)} < 0) \rightarrow 1 + a^n > 2a^{n-1}. \quad (\text{A}\cdot\text{7})$$

Putting  $a = 1 - \gamma$  (assuming  $\gamma$  is small), we obtain

$$4 > n \geq 1 \quad (\text{A}\cdot\text{8})$$

from (A·6) and (A·7). The case  $n=1$  is easily shown to be satisfied.

In this case, the possible periodicity of simple subcyclic motion is 2, 4 and 6.

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