A Simulation Study of Large Scale Swarms

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Abstract: Reynolds’s boid model (1987) is one of the first models to successfully produce swarming behavior. However, the size of the swarms was limited to a few hundred individuals. The present study is reformulated based on the original model to simulate the swarm size up to 500,000 individuals by utilizing the GPGPU technique. The preliminary study reports a qualitative change in swarm formation in larger swarms. We focused on a particular set of parameters that enables complex swarming behavior and showed: (i) a new classification of swarm groups are tested, (ii) a correlation strength gradually increases as the size of swarm increases, (iii) 10 percent of the members are exchanged but the size of the swarm is kept constant as a large swarm, and (iv) the maximum size of the swarm grows with a power law as a function of the system size.

Keywords: Selected keywords relevant to the subject.

1. INTRODUCTION

Studying collective behavior by using a simple abstract model is important in order to understand the mechanism of swarming. When we want to know the larger swarm’s behavior, simple constructive models are particularly useful. Reynolds’s boid model is a typical simulation model [1]. It is one of the oldest and most successful models to simulate collective swarm behavior and has three simple forces: attractive, repulsive and alignment. Since his pioneering work, different realizations of the boid model have been proposed and discussed. For example, Vicsek’s model [2] was concerned with the alignment rule to show rich complex swarming behavior. The model has been studied in the sense of statistical physics [3]. Couzin, 2002 [4] used the differential equation corresponding to Reynolds’s original model and classified the swarming behavior into four phases. Other simulation models also considered the three simple rules (or variations) and took environmental factors into account as noise terms (Aoki, 1982[5]; Okubo, 1986[6]; Reynolds, 1987[1]; Huth and Wissel, 1992[7]; Couzin, 2003[8]).

Other than simplifying the generalization of a boid model, a new computational method called GPGPU has been developed to run large-scale simulations. For example, Sampson et al. (2009) elaborated a large scale simulation by using a new parallelization with GPGPU. We adopted a simple parallelization method by using a local neighbor matrix associated with each agent in order to update their states (see e.g., [9]). Then we used a GPGPU system to simulate a large scale model with parallelized differential equations. The details of the model and parallelizing method will be given in the below sections.

2. REYNOLDS’ BOID MODEL

Reynolds’s boid model is characterized by three distinct rules; attraction, repulsion, and alignment, i.e., (i) each boid is attracted to but (ii) also repulsive to the center of mass of the swarm, and (iii) they mutually align their headings in a parallel way. We translate them into a differential equation based on the velocity of each boid:

$$\Delta \vec{v}_i = W_{att} \cdot (\vec{x}_i - \frac{\sum_{j} s_{att} \vec{x}_j}{n_{att}}) + W_{rep} \cdot (\sum_{j} s_{rep} \frac{(\vec{x}_i - \vec{x}_j)}{|\vec{x}_i - \vec{x}_j|}) + W_{ali} \cdot (\vec{v}_i - \frac{\sum_{j} s_{ali} \vec{v}_j}{n_{ali}})$$

The position of each void is updated by the computed velocity $$\Delta \vec{x}_i$$ iteratively. The attraction and repulsion terms are represented by the first and second term, respectively. Each rule has an interaction range around each agent and is denoted by $$s_{att}$$, $$s_{rep}$$, and $$s_{ali}$$, respectively. In the equation, we also introduce the amplitudes of those interactions by $$W_{att}$$, $$W_{rep}$$, and $$W_{ali}$$, respectively. In order to avoid the excess speeding up or down, we bound the amplitude of speed in between $$V_{min}$$ and $$V_{max}$$.

3. GPU IMPLEMENTATION

The computational cost of the original boid model can be estimated as $$O(n^2)$$ as the worst case while taking the total number of individuals as $$n$$. When parallelizing the computational steps by using the GPGPU method, we don’t have to consider every possible pair of agents. Fortunately, our boid model has a finite interaction range, so that the computation cost can be reduced. We divide the whole space into a small size mesh (200 x 200 x 200) and each individual will be registered to the corresponding mesh ID. In this way, we divide the entire simulation process into independent ones [9]. By using this method, we achieved approximately 500,000 agents by 1 fps in our simulations.

The 3-dimensional periodic boundary condition was applied here, but this is an unrealistic condition. The actual parameters used in the simulation are listed in Table 1.

† Yhoichi Mototake is the presenter of this paper.
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Table 1 Parameters of this simulation

4. RESULTS : COEXISTENCE OF DIFFERENT SWARMING PATTERNS

4.1. observations

Couzin (2002) studied the original boid model, changing the interaction range of attraction and alignment [4]. With the total number of individuals equal to 100, he classified rather stable swarming motion into four types: swarming, torus, a dynamically parallel motion (all individuals move in the same direction without spatial coherence), and a highly parallel motion (all individuals move in the same direction).

Following Couzin’s work, we were mainly concerned with using a large number of individuals to find a new type of swarm movement on a large scale. We ran a simulation with randomly distributed individuals. The parameters of the simulation were explored in the area of the dynamic parallel phase in Couzin’s work. A criterion for finding a parameter set was to have an unstable swarm in the lower individual scale. We then increased the number of individuals to see how swarm organization develops. In order to maintain a constant density of the number of individuals, we expanded the space size as we increased the population size.

In case of 16,384 individual boids (Figure 1), a single large swarm was discovered to exist rather stably. Stretched filaments or snake-like patterns (hereafter called fragmented filaments) coexist with the large swarm. By further increasing the number of individuals, swarm dynamics become radically complex when the individual size equals 131,072 (see Figure 2). Some swarms are composed of a very large number of individuals and accompanied by small size filaments and unstable swarms. The larger swarms were moving very slowly. On the other hand, the filaments were moving rapidly, changing their patterns in unexpected ways. Furthermore, by increasing the individual size up to 524,288, larger size swarms structures were observed (see Figure 3). We used a slightly larger density size to produce Figure 3 compared with Figure 1 and Figure 2.

4.2. Analysis

The stability of swarm dynamics is dependent on the swarm size and on the neighboring swarms. An initially prepared large-scale swarm will be broken up into smaller filaments and swarms; however, they will re-organize into large swarms after a while. This collapse and re-organization process will be repeated i.e., swarms can exist by exchanging individual boids. These observations suggest that (i) we need to refine the definition of a single swarm, and (ii) an order parameter is required to characterize swarm dynamics.

Below, we show that DBSCAN is the most effective way to cluster different swarms. By elaborating some clustering techniques, we could finally identify distinct swarms. After identifying each swarm, we computed an average velocity and the fluctuation of velocities within a single swarm. We show that the correlation length goes up when the size of swarms goes up.

4.2.1. Clustering Swarm

We first studied the number density distribution (n.d.d.) of individuals to distinguish qualitatively different swarms. By reducing the entire grid into a $60 \times 60 \times 60$ coarse grained mesh, we computed the n.d.d. on this coarse grained mesh, as shown in Fig.4. This n.d.d. be-

Fig. 1 The simulated result with 16,384 individuals. A single large size swarm exists across the periodic boundary. Some filament patterns are visible.
comes stable at least after 10,000 steps, which is regarded as a combination of a power and the Gaussian distribution. By fitting the distribution with $Ax^{-\gamma} + Be^{-(x-b)^2/2\sigma^2}$, we can say that the distribution consists of large scale swarms centering around the Gaussian distribution and smaller scale swarms, including filaments, obeying a power law distribution.

A clustering technique called density-based spatial clustering (DBSCAN) ([10]) was found to be the most effective. It is more intuitive and precise than, for example, standard k-means clustering. While k-means classifies based on the distance between individuals and the center of the mass, DBSCAN classifies based on the neighboring individuals connected within a certain distance. We thus used it to distinguish swarms by using the standard deviation $2\sigma$ from the fitted Gaussian curve as the critical threshold. The additional parameter used by DBSCAN is a minimum cluster size, which is set at 2.

In the analysis below, we only focused on the case of 131,072 individuals. The system slowly relaxes to an equilibrium state, and the statistical properties seemed to stabilize after 10,000 steps. In Figure 5, we show the result of clustering at 20,000 steps. From this result, we see that larger swarms were extracted successfully but smaller swarms were not clustered. Individuals in the internal region of swarms were clustered as a member of the same swarm, but those on the surface of the swarm were not. Indeed, they temporally come and go from a swarm too frequently to be classified as the member.

Figure 6 is the distribution of the distinct swarm based on the distance between individuals and the center of the mass, DBSCAN classifies based on the neighboring individuals connected within a certain distance. We thus used it to distinguish swarms by using the standard deviation $2\sigma$ from the fitted Gaussian curve as the critical threshold. The additional parameter used by DBSCAN is a minimum cluster size, which is set at 2.

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sizes. The larger swarms (around $10^4$ individuals) and the smaller swarms (around $10^3$ individuals) were separated to between 1,000 and 10,000 individuals. Note that this swarm size distribution is only obtained after applying DBSCAN.

It should be noted that even for the same swarm, the surface individuals can be replaced constantly. Figure 7 shows that at least 10 percent of the individuals were replaced with others (left figure), but its size deviation remained constant.

4.2.2. Fluctuation, velocity and size

To quantitatively characterize the different swarm, we computed the average velocity $\frac{1}{N} \sum_{i=1}^{N} \vec{v}_i(t)$. Figure 8 denotes the relationship between the size of swarms and their velocities. The larger swarms did not move significantly, but the smaller swarm had larger velocities, and its distribution had two peaks. We noticed that this velocity distribution bifurcates from a single peak to double peaks between a size of 10 and 100 individuals. Smaller swarms tended not to belong to any of the swarms and rapidly moved around.

Then we compute the spatial fluctuation of velocities within a same swarm $\frac{1}{N} \sum_{j \in S} \delta \vec{v}_i \cdot \delta \vec{v}_j$ where $\delta \vec{v}_i = \vec{v}_i - <v>_{ave}$ was computed (Figure 10). The spatial fluctuation can be related to susceptibility that measures potential response against external disturbance. The larger the swarm’s susceptibility, the more sensitive the swarm is to perturbation. The larger fluctuation expresses less stability, but it is more adaptive and evolvable in the evolutionary context.

We also measure the correlation length between velocities within a same swarm by $C(r) = \frac{\sum_{i,j} \delta \phi_i \cdot \delta \phi_j \cdot \delta(r-r_{ij})}{\sum_{i,j} \delta(r-r_{ij})}$ where $\delta \phi_i = \frac{\delta v_i}{\sqrt{\sum_{k} (\delta v_k)^2}}$. It decays exponentially and we take the first zero crossing point $C(r_0) = 0$ as a correlation length $r_0$. Figure 9 represents the relationship between the size of swarm and the correlation length. It grew gradually (logarithmically) as the size increased. This means that larger swarms become more coherent than the smaller swarms; meanwhile, the susceptibility decreases as the swarm size increases. Apparently, there exists a global correlation among swarms.

In the end, Figure 11 shows the largest swarm found in the given system size. Since the size of the largest swarm increases by the power law, we claim that (i) a larger size swarm can emerge by increasing the number of individuals, and (ii) it’s max size grows with a power law (Figure 6). Since the power law implies that there exists a global correlation among swarms, we speculate that the swarms are maintained by interacting with others, i.e., smaller ones are filled from the larger swarms, and the larger swarms are re-organized from smaller ones. Therefore, a single swarm cannot be extracted alone.

5. SUMMARY AND DISCUSSION

Due to the parallelization method using GPGPU, we have successfully simulated larger scale swarms. Our re-
results imply that the original Reynolds’ model can have much more complex swarming by merely increasing the number of individuals for some parameter regions at least. Furthermore, from quantitative analysis, we re-

![Relation of size and Correlation length](image1)

*Fig. 9* A correlation length calculated for each swarm size.

![Relation of size and susceptibility](image2)

*Fig. 10* The relationship between susceptibility and swarm size. Midsize swarms had relatively large susceptibility but larger swarms had smaller values.

![Relation of simulation size and max bold size](image3)

*Fig. 11* The largest swarm size found in the different system size (i.e., the total number of individuals).

vealed that different size swarms coexist together and actually they are maintained by replicating individual members.

There exist two qualitatively different swarms: one with a power law distribution and one with the Gaussian distribution with respect to the number densities. The former corresponds to smaller swarms and the latter one to larger swarms. We also found that smaller swarms move at much higher speed than larger ones. We speculate that this tendency is caused by the fact that larger swarms can only exist by constantly exchanging with other swarms and cannot keep its member by themselves. This is also confirmed by the power law distribution of the swarm sizes and by simulating the system from an initial single swarm. The present study reveals the unstable nature of the original Reynolds’s model by increasing the system size. We have already confirmed more varieties in swarm behaviors with different parameter settings. The present study only deals with a special set of parameters, but we believe that the present results implies how larger swarms behave when they are organized and developed in a general sense.

**REFERENCES**