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NP-completeness of kSAT and multifractals

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Abstract

The geometrical structure of a formal language class is studied in relation with its time-complexity. A typical NP-complete problem, kSAT, is discussed by visualizing its problem space and a strict connection is made between the self-similarity and the time-complexity of the languages by constructing adequate iterated function systems (IFSs). There exist IFS classes which generate whole satisfiable Boolean expressions embedded on a unit hyper-cube. Our numerical results for the Hausdorff dimension of kSAT suggest the difference of IFS classes for 2 and 3SAT.

The decision problem of satisfiability of Boolean expressions in k-conjunctive normal form (kSAT) is a typical NP-complete problem. It is known that 2SAT is in class P and 3SAT is in class NP. The relationship between class P and NP is the most important research subject in computational complexity theory. In this paper, we introduce the concepts of fractals to classical complexity theory with kSAT taken as an example. Recently kSAT has been studied by massive computer experiments and some new evidences of computational complexity have been revealed [2]. Here we embedded the whole Boolean expressions in kCNF on a k-dimensional unit space with the method of interval real arithmetic [1]. Next we solved all problems with n variables and plot satisfiable expressions as points in the unit space. These visualizations enable us to observe the spatial pattern of kSAT on the k-dimensional space. We call the set of the points corresponding to satisfiable Boolean expressions in kCNF with n variables simply S_n^k. Then the decision problem "Let F be a Boolean formula in kCNF, then $F \in k$SAT?" is equivalent to the decision problem "Let x be a point in an unit space, then $x \in S_n^k$?" Taking a point in a unit space as an instance of kCNF, we can solve the problem to decide whether the point is inside or outside of S_n^k. Thus we are interested in the shape of S_n^k.

The spatial pattern of S_n^2 on a unit square for $n = 1, 2, 3, 4$ and S_3^3 on a unit cube are given in Fig. 1. S_n^2 are symmetric with respect to the diagonals and have apparent self-similarity. By taking the two-dimensional vertical section with respect to an axis, we get the two-dimensional geometrical structure of S_3^3. The section face of S_n^3 including the origin is S_n^2 which corresponds to 2SAT in class P. We can easily reconstruct the iterated function systems (IFSs) which generate 2SAT. We however face difficulties to reconstruct them for 3SAT. It is thought that S_n^3 has more complex geometrical structure than S_n^2. We introduce two types of IFS here. One is monotone IFS, which has simple nested structure like tree, and the other is recurrent IFS, which has complex nested structure with loop [4]. We attempt to compute the Hausdorff dimensions of kSAT with the box-counting method. The Hausdorff dimension $\dim_H$ can

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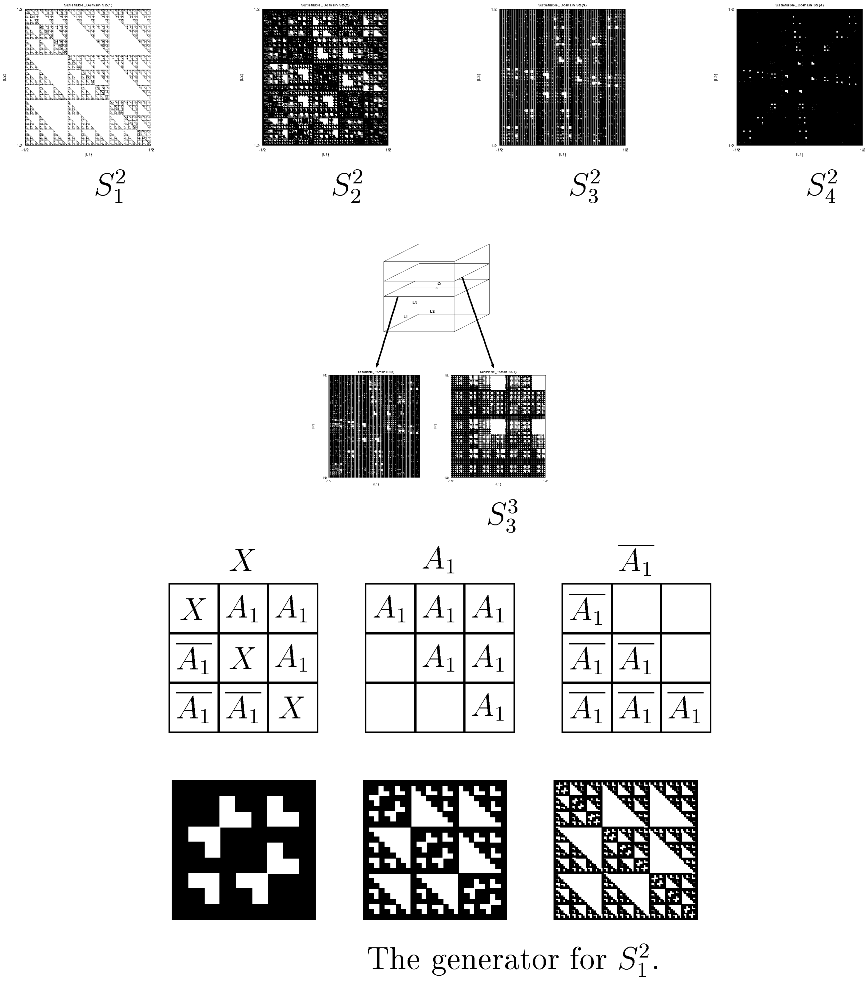

Fig. 1. Black points in figures are corresponding to the satisfiable expressions in kCNF with n variables. By increasing the number of the variables n, we obtain more complex patterns. Section faces of S_3^3 give heterogeneous patterns compared with S_3^2. We can regenerate S_n^2 with simple IFS.

be analytically calculated for the simple self-similar sets generated by monotone IFS [4],

$$\dim_{\mathrm{H}}(S_n^k) = \frac{\log[(2n+1)^k - \{(n+1)^k - 1\}]}{\log(2n+1)}. \qquad (1)$$

In the case of 2SAT, our experimental result does well fit the analytical values of the Hausdorff dimension. However in the case of 3SAT, it breaks (Fig. 2).

With these results we conjecture that S^2 is the limit set of monotone IFS, and S^3 is not. It is supposed that S^3 can be generated by more complex IFSs such as the recurrent IFS, the IFSs which do not satisfy the open set condition, or the IFSs which consist of some non-linear contraction mapping. The complexity classes therefore can be naturally compared to self-similarity classes. If we have an IFS coding which maps a language in NP to a simple self-similar set, it is regarded that such IFS code should be equivalent to Turing polynomial reduction for the language in class NP. A new time-complexity measure is also given in terms of the Hausdorff dimension of formal languages, and the model presented here can be one of the computation theories on R [3]. Introducing

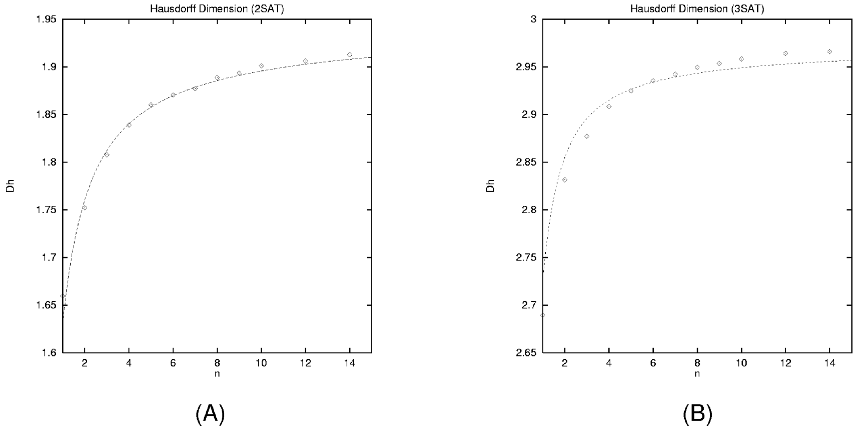

Fig. 2. The box-counting dimension of (A) S_n^2 and (B) S_n^3 as a function of the partition size n. The points indicate numerical estimation of the Hausdorff dimension by box-counting and the dotted lines the analytical values by Eq. (1).

some kind of measure into this problem space, we can calculate $f(\alpha)$-spectrum and get more detailed property of NP-complete structure, but that is future work.

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